True triaxial strength and deformability of the German Continental Deep Drilling Program (KTB) deep hole amphibolite

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Abstract. We designed and fabricated a true triaxial loading system and used it to determine deformational and strength characteristics of the amphibolite penetrated by the superdeep hole drilled in the Bohemian massif of southeastern Germany under the German Continental Deep Drilling Program (KTB). Amphibolite is found between 3200 and 7300 m and thus the dominant rock in this 9100-m boring. Our loading system enables the application of three unique principal stresses to a rectangular prismatic rock specimen. During a test we maintained the least principal ($\sigma_3$) and the intermediate ($\sigma_2$) stresses constant and increased the major principal stress ($\sigma_1$) until brittle failure occurred, in the form of a fracture steeply dipping in the $\sigma_2$ direction. Typically, for the same $\sigma_3$ level the amphibolite compressive strength increased substantially with the magnitude of $\sigma_2$, demonstrating the inadequacy of Mohr-like failure criteria that ignore the effect of the intermediate principal stress on rock strength. We found that a general criterion for the amphibolite could be expressed in the form of a power function relating the octahedral shear stress at failure to the mean normal stress acting on the plane containing the fracture. With respect to deformation, we established that for the same $\sigma_3$, the onset of dilatancy increases significantly with the magnitude of $\sigma_2$. Thus the intermediate principal stress appears to extend the elastic range of the stress-strain behavior for a given $\sigma_3$ and hence to retard the onset of the failure process. Scanning electron microscopy observations of the failure process reveal that microcracks develop mainly parallel to $\sigma_2$ direction, as the intermediate stress grows beyond $\sigma_3$, localizing in close proximity of the eventual main fracture.

1. Introduction

The German Continental Deep Drilling Program (KTB) was designed to study the properties and processes of the lower continental crust by means of deep boreholes. Under this program a superdeep hole, reaching a final depth of 9100 m, was drilled in the Bohemian massif of southeastern Germany. Among the main themes of the KTB borehole project was the study of basement properties and deformation mechanisms, and the measurement of the in situ stress [Finnermann and Lauterjung, 1997]. The objective of our research was to examine the deformational and strength properties of the amphibolite penetrated by the KTB hole (henceforth called KTB amphibolite) under the most general (or true triaxial) in situ stress conditions and to provide rock strength data needed for a better estimate of the crustal stress field.

For the purpose of estimating the crustal stress field through the KTB hole, scientists were compelled, owing to hostile borehole conditions, to employ a modified version of the hydraulic fracturing method. In addition, they were also limited to just two tests, at 6 and 9 km depth, which yielded estimates of the least principal stress magnitude alone [Engesser et al., 1993; Zoback and Harjes, 1997]. These tests were complemented by stress approximations interpreted mainly from logged borehole breakouts between 3200 and 6800 m [Brudy et al., 1997], a depth range within which amphibolites are the dominant rock type [Dykster et al., 1995]. The use of logged borehole breakout spans to estimate the major horizontal principal stress when the vertical and least horizontal stresses are independently known had been proposed earlier by Vernik and Zoback [1992]. Their model assumes that the stress condition at the points where breakouts intersect the borehole is one satisfying the strength criterion of the rock. Vernik and Zoback suggested that the appropriate stress condition for this case should be obtained under true triaxial stress conditions ($\sigma_1 > \sigma_2 > \sigma_3$), since all three principal stresses at the borehole wall are noticeably different. For lack of empirical strength data, Brudy et al. [1997] obtained estimates of the major horizontal principal stress around KTB based on a theoretical true triaxial stress formulation. It was felt, however, that an experimentally obtained criterion was necessary in order to verify these initial approximations.

Consequently, we embarked on a comprehensive study of the true triaxial mechanical behavior of the KTB amphibolite, emphasizing strength and deformability. We first designed and fabricated a true triaxial testing system capable of applying three independent and mutually perpendicular compressive loads to prismatic rock specimens of rectangular shape and tested it by conducting an initial comprehensive series of experiments in Westerly granite [Haimson and Chang, 2000]. We then used the apparatus to carry out two extensive series of true triaxial tests in KTB amphibolite. In the first series of tests, jacketed specimens were loaded to failure under the
most general state of stress \((\sigma_1 > \sigma_2 > \sigma_3)\), simulating in situ conditions down to 10 km. The results of these experiments provide fundamental data on the mechanical behavior of the amphibolite and are fully described in this paper. A second series of tests was conducted specifically for the purpose of estimating in situ stress magnitudes at KTD from borehole breakout dimensions and involved a special case of amphibolite strength and deformability. In these tests, one of the principal stresses was applied by pressurized fluid directly in contact with a pair of opposed specimen faces that was left unjacketed. The intent was to simulate borehole wall rock, which is not only subjected to in situ stresses but is also exposed to borehole fluid. The results of this series of tests will be the subject of a follow-up publication.

The present paper begins with a discussion of the most common strength criteria for rocks, emphasizing the need for one that considers the effect of both the least and the intermediate principal stresses. That is followed by a description of the equipment used and the laboratory experimental setup and procedure. Finally, test results in terms of rock strength are presented, leading to a new true triaxial strength criterion. Also discussed is the dependence on the intermediate principal stress of the failure plane dip angle, the stress-strain relationship, the onset of dilatancy, and the micromechanics of failure.

2. Rock Compressive Strength: Conventional Triaxial or True Triaxial?

Rock strength can generally be defined in terms of the critical state of stress condition that leads to failure (or fracture). Knowledge of this mechanical property is crucial in engineering design of structures as well as in geophysical research such as the study of fault and earthquake mechanics and the determination of in situ stress (e.g., hydraulic fracturing, borehole breakouts, drilling-induced cracks). Strength criteria are often expressed in terms of the maximum principal stress \(\sigma_1\) that rock can sustain for known magnitudes of the other two principal stresses \((\sigma_2, \sigma_3)\). In its most general form this can be expressed as \(\text{Scholz, 1990, p. 13}\)

\[
\sigma_1 = f(\sigma_2, \sigma_3),
\]

where \(f\) is a function to be determined theoretically or empirically. However, in the most commonly used criteria the effect of the intermediate principal stress \(\sigma_2\) is neglected, leading to a simpler relationship:

\[
\sigma_1 = f(\sigma_3).
\]

One such simplified criterion is the linearized Mohr criterion (also called Mohr-Coulomb), which in terms of the two extreme principal stresses takes the form \(\text{Jaeger and Cook, 1979, p. 96}\)

\[
\sigma_1\left(\sqrt{\mu^2 + 1} - \mu\right) = 2\sigma_3 + \sigma_1\left(\sqrt{\mu^2 + 1} + \mu\right),
\]

where \(S_1\) and \(\mu\) are two material properties termed cohesion and coefficient of internal friction, respectively. Since this criterion is simple and relatively easy to obtain from a series of conventional triaxial compression tests on cylindrical samples in which \(\sigma_1 > \sigma_2 = \sigma_3\), it has gained widespread acceptance among rock mechanics practitioners.

Another well-known strength criterion in which the effect of \(\sigma_2\) is ignored is derived from the Griffith [1924] theory of unstable crack extension. This criterion assumes that rock failure occurs upon the initiation of propagation of a most critically oriented preexisting crack. The Griffith criterion in the compressive range of \(\sigma_1\) and \(\sigma_2\) is expressed as

\[
(\sigma_1 - \sigma_1)^2 = 8T(\sigma_1 + \sigma_1),
\]

where \(T\) is the rock tensile strength. This theoretical criterion was later modified by McClintock and Walsh [1962], who incorporated the effect of frictional sliding along the closed Griffith crack under compression. This modification coincidentally led to a criterion similar in principle to that of Mohr-Coulomb:

\[
\sigma_1\left(\sqrt{\mu^2 + 1} - \mu\right) = 4T + \sigma_1\left(\sqrt{\mu^2 + 1} + \mu\right),
\]

where \(\mu\) is the coefficient of sliding friction along the closed Griffith crack, a material property.

More recently, Horii and Nemati-Nassri [1985], Ashby and Hallam [1986], Ashby and Sammis [1990], among others, used a damage mechanics approach to describe rock failure. The strength criteria suggested by these studies are based on the assumption that, under compressive stress, wing cracks propagating below a given volume of rock grow, interact with other cracks, and ultimately coalesce leading to a final macroscopic failure. One simplified criterion, which has an explicit form, is that derived by Ashby and Sammis [1990]:

\[
\sigma_1 = C_0 + C_0\sigma_0,
\]

where \(C\) is a constant depending on dimensions and density of initial microcracks, and on the coefficient of sliding friction along such cracks, and \(C_0\) is the rock uniaxial compressive strength.

The exclusion of \(\sigma_2\) as a parameter affecting rock strength was not shared by Nadai [1950], Freundenthal [1951], Drucker and Prager [1952], Bretser and Fiser [1957], and others, who included all three principal stresses in their criteria of failure for other materials, such as soils and concrete. Nadai [1950, p. 231] described the limiting stress of brittle materials by modifying the von Mises [1913] yield criterion for ductile metals that had been derived based on the distortional energy concept. Nadai suggested that the mechanical strength of brittle material is reached when the octahedral shear stress and octahedral normal stress (the mean normal stress) are related by a monotonically increasing function \(f\), i.e.,

\[
\tau_{\text{oct}} = f(\sigma_{\text{oct}}),
\]

where

\[
\tau_{\text{oct}} = \frac{1}{2}(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)^{1/2}
\]

and

\[
\sigma_{\text{oct}} = \left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{3}\right)^{1/3}.
\]

A simplification of this criterion in the form of a linear relationship between the octahedral shear and normal stresses was suggested by Freundenthal [1951] and Drucker and Prager [1952] for concrete and soil, respectively.

\[
\tau_{\text{oct}} = c_1 + c_2\sigma_{\text{oct}},
\]

where \(c_1\) and \(c_2\) are positive material constants.

Murrell [1963] and Handin et al. [1967] were among the first to note the effect of the intermediate principal stress on rock strength. They observed that the strength of Carrara marble and Solenhofen limestone, respectively, was consistently higher when subjected to conventional triaxial extension \((\sigma_1 > \sigma_2 > \sigma_3)\) than under conventional triaxial compression \((\sigma_1 > \sigma_2 = \sigma_3)\). Their observations prompted several researchers to seek more general criteria, which could describe
rock strength under true triaxial compressive stress conditions \(\sigma_1 \geq \sigma_2 \geq \sigma_3\). Murrell himself suggested such a criterion for Carrara marble, based on his experimental observations, by extending the initial Griffith criterion given by equation (4) to include the intermediate principal stress \(\sigma_2\):

\[
\tau_{oc} = 87 \sigma_{oc}.
\]  

(9)

However, to our knowledge this criterion has not been verified experimentally.

A theoretical approach to estimating the strength of rock that is subjected to true triaxial stress conditions was suggested by Wiebols and Cook [1968]. Their hypothesis was that rock failure occurs when the total effective shear strain energy summed over all preferentially oriented sliding cracks in a rock body reaches a critical value dependent on the coefficient of sliding friction \(\mu\) and the uniaxial stress of rock \(\sigma_u\). Wiebols and Cook presented their criterion for individual \(\mu\) values in the form of functions of curves in the \(\sigma_1/\sigma_u\) versus \(\sigma_2/\sigma_u\) domain, each curve for different magnitudes of \(\sigma_1\). Their conclusion was that as \(\sigma_1\) is raised from \(\sigma_2 = \sigma_1\) to \(\sigma_2 = \sigma_u\), the strength \(\sigma_1\) for a constant \(\sigma_2\) first increases, reaches a maximum at some intermediate value of \(\sigma_2\), and then decreases to a value greater than the conventional triaxial equivalent when \(\sigma_2 = \sigma_u\). The Wiebols and Cook criterion has not been widely used, partly because it requires prior knowledge of the sliding coefficient of friction \(\mu\), a property that is not given to direct measurement. However, their theory was utilized by Brudy et al. [1997] to obtain an estimate of the in situ maximum horizontal stress from borehole breakout dimensions at the KTB site.

The first extensive true triaxial compressive tests in rocks were conducted by Mogi [1971]. He designed a polyaxial loading apparatus, which enabled the application of three independent and unequal orthogonal loads to each pair of faces of a rectangular prismatic specimen. His tests in Dunham dolomite unequivocally demonstrated the strong dependence of rock strength on \(\sigma_2\) for a given \(\sigma_1\) magnitude, a dependence similar in manner to that predicted theoretically by Wiebols and Cook [1968]. Mogi attempted to find a simple relationship that would satisfy all his experimental results. He noted that the Nadai [1950] criterion, for which the independent variable is the three-dimensional mean normal stress \(\sigma_{oc}\) (equation (7)), was adequate for characterizing rock yielding that occurs over the entire volume of rock. He discovered, however, that in his tests brittle failure occurred along a steeply inclined plane, striking in the \(\sigma_2\) direction. Hence he deduced that the mean normal stress contributing to the creation of the failure plane was not \(\sigma_{oc}\), but its two-dimensional representation: \(\sigma_{oc} = (\sigma_1 + \sigma_2)/2\). Consequently, Mogi derived a new strength criterion that fit all his true triaxial test results in Dunham dolomite:

\[
\tau_{oc} = f(\sigma_{oc}),
\]  

(10)

where \(f\) is a monotonically increasing function. He later verified this criterion for several other rock types.

Although Wiebols and Cook [1968] and Mogi [1971] demonstrated by independent means that the intermediate principal stress has a major effect on rock strength, their work was largely ignored for over 20 years. The reason for this lies mainly, we suspect, in the experimental difficulty of determining rock true triaxial strength, vis-à-vis the simplicity of testing cylindrical specimens inside a pressure vessel, as required for obtaining a Mohr-type criterion.

Recently, the interest in the true triaxial strength of rocks has been rekindled in part by the need to employ an appropriate failure criterion in order to relate quantitatively borehole breakout dimensions to the prevailing in situ stress [Vernik and Zoback, 1992]. Haimson and Chang [2000] designed a true triaxial testing apparatus and conducted an exhaustive series of tests in Western granite. Their results largely confirm those obtained by Mogi [1971] for Dunham dolomite, namely, that the intermediate principal stress strongly affects rock strength and should therefore be explicitly included in the strength criterion formulation. Haimson and Chang also showed that in Western granite \(\sigma_2\) affects the failure plane dip angle (higher \(\sigma_2\) yields steeper dip) and the extent of linear elastic deformation (higher \(\sigma_2\) retards dilatancy onset).

3. The KTB Amphibolite

The amphibolite samples used in our experimental study were prepared from core (234 mm diameter) extracted from the main KTB borehole at a depth of 6355-6360 m. Generally, it is a massive metamorphic rock, which at that depth has little or no foliation [Dyuster et al., 1995]. We analyzed a thin section of the amphibolite using a point-counting method and derived its mineral composition as being 58% amphibole (mainly hornblende, average grain size 0.4 mm), 25% plagioclase (average grain size 0.2 mm), 5% garnet, 2% biotite, and 7% minor opaque minerals. The amphibole, which is the dominant mineral, is evenly distributed and randomly oriented (Figure 1).

We measured some basic physical properties of the amphibolite core available to us (Table 1). Density was determined from measurements of mass and volume of prepared dry specimens. Effective porosity was obtained using a Royles' law porosimeter. Uniaxial (unconfined) compressive strength was determined by axially loading cylindrical specimens (25 mm diameter and 51 mm long) to failure in a calibrated loading machine. Table 1 also displays independently measured physical properties representative of the entire amphibolite core recovered from the KTB deep hole. Our results, representing a limited range of core depths, compared well with the previously obtained properties. This suggests general uniformity of amphibolite physical properties throughout the depth range in which it was encountered by the KTB deep hole. The low porosity and permeability, and the high density and compressive strength, reflect the amphibolite tightly interlocked fabrics (see also Figure 1).

Some additional mechanical characteristics of the KTB amphibolite were established by Vernik et al. [1992], who carried out conventional triaxial tests on cylindrical specimens cut in two different axial directions, 90° and 30° to foliation. The amphibolite showed very little anisotropy and behaved elastically almost up to the point of failure.

For true triaxial compressive testing, we prepared rectangular prismatic specimens (19x19x38 mm), which were surface ground so as to obtain dimensions within 0.2 mm from the prescribed size, with a minimum of parallelism and orthogonality offsets. Attempts were made to leave out any visible fresh cracks observed on the core, which might have been drilling-induced or the result of stress relief during coring. Each specimen was prepared with the long dimension aligned with the axis of the core. The specimens were oven-dried at 40°C for at least 24 hours before testing.
Figure 1. A thin section of unstressed KTB amphibolite showing the dominance of evenly distributed amphibole crystals (gray) exhibiting randomly oriented cleavages.

4. Experimental Setup and Procedure

The laboratory tests described in this paper were conducted using the University of Wisconsin true triaxial loading system, which consists of a polyaxial pressure vessel inside a biaxial loading apparatus (Figure 2). Three independent and mutually orthogonal pressures are generated in this system and applied to a rectangular prismatic specimen. The biaxial apparatus facilitates the application of two pressures, one in the axial ($\sigma_1$) and the other in one of the two lateral directions ($\sigma_3$) of the specimen. These two pressures are transmitted from the biaxial cell to the rock specimen via two perpendicular pairs of pistons mounted in the pressure vessel. The third pressure ($\sigma_3$) is applied directly to the second pair of specimen lateral faces by the confining hydraulic pressure inside the pressure vessel. The two loads applied by the biaxial apparatus are monitored by calibrated strain gages mounted on the respective pistons. The load measurements are internal to the pressure seals. The pressure in the third direction is measured by a pressure transducer installed in line with the confining pressure. The loading system was thoroughly cali-

Table 1. Some Physical Properties of the KTB Amphibolite

<table>
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<th>Property</th>
<th>Our Measurements*</th>
<th>Previous Measurementsb</th>
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<td>Number of Tests</td>
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<td>Density, kg/m$^3$</td>
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<td>Porosity, %</td>
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<td>0.70 (±0.33)</td>
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<td>Permeability, m$^2$</td>
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<tr>
<td>Uniaxial compressive strength, MPa</td>
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<td>164 (±9)</td>
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</table>

* Tests on core samples extracted from 6355-6360 m.

b Tests on core samples from the full length of the recovered amphibolite core. For density measurements use was also made of cuttings.

NA, not available.
brated using a strain-gaged aluminum sample of known elastic properties. A detailed description of the calibration procedures and results is given by Haimson and Chang [2000]. The maximum stresses that this loading system can apply to a rock specimen of dimensions $19 \times 19 \times 38$ mm are 1600 MPa in the two piston-loading directions and 400 MPa in the third direction. This high load capacity renders the equipment capable of testing very strong rocks to failure under simulated in situ stress conditions of considerable magnitude.

Strains in the three principal stress directions are monitored during testing. Strain measurements in the $\sigma_1$ and $\sigma_2$ directions are made using properly oriented strain gages bonded to the piece of specimen lateral faces subjected to $\sigma_3$ (Figure 3a). Strain in the $\sigma_3$ direction is measured using a beryllium-copper strain gaged beam mounted parallel to the $\sigma_1$ faces (Figure 3b). The ends of this beam are fixed, while the center is pushed outward by a pin affixed to the exposed face of the specimen. During a true triaxial test the beam flexes as the pin compresses the beam center. Strain gages mounted near the beam ends record its bending and enable the strain monitoring in the $\sigma_3$ direction.

Specimen sides that are to be subjected to piston loading are placed between pairs of metal anvils (16 mm thick) that are identical in cross-sectional dimensions to those of the specimen faces. The anvils serve to uniformly distribute the loads transmitted through the loading pistons to the specimen (Figure 3). A copper shim, 0.03 mm thick, is inserted between rock specimen and metal anvils to minimize friction. Stearic acid is smeared over the contact faces between the copper shim and anvils as a lubricant [Labasc and Bridell, 1993]. The specimen is then coated with a thin layer of polyurethane to prevent permeation of the confining fluid applying $\sigma_3$. The instrumented specimen is inserted into the polyaxial pressure vessel using a special guiding device.

True triaxial testing procedure consisted of simultaneously raising all three principal stresses at a constant rate until $\sigma_1$ reached its preset value. Thereafter, the other two principal stresses ($\sigma_2$ and $\sigma_3$) were increased at the same rate until $\sigma_2$ reached its predetermined magnitude. From this point, $\sigma_2$ and $\sigma_3$ were kept constant and $\sigma_1$ alone was raised, by controlling the least principal strain ($\varepsilon_3$), at a rate of $5 \times 10^{-6}$ s$^{-1}$, until the specimen failed. This loading mode enabled the specimen to fail stably and to continue to carry load even after $\sigma_1$ reached its peak value. Unloading was typically carried out only after $\sigma_1$ decreased $\sim 10\%$ of its peak level. Using the three-stage loading path, the order of stress magnitudes, $\sigma_1 \geq \sigma_2 \geq \sigma_3$, was maintained throughout the experiment.

5. True Triaxial Compressive Strength of KTB Amphibolite

The first stage of our laboratory testing consisted of conducting several uniaxial compression experiments ($\sigma_1 > \sigma_2 = \sigma_3 = 0$) in which cylindrical specimens of KTB amphibolite were loaded until failure. The mean uniaxial compressive strength based on these tests is $164 \pm 9$ MPa, which is somewhat lower than that determined by Röckel and Nataf [1995] based on a much larger sampling of data ($189 \pm 67$ MPa), but is certainly within its standard deviation.

In the second stage of our laboratory work we carried out conventional triaxial experiments ($\sigma_1 > \sigma_2 = \sigma_3 > 0$) on rectangular prismatic specimens inside the true triaxial loading apparatus. In these tests, uniform fluid pressure was applied to all lateral faces of the specimen and held constant while $\sigma_1$ was raised (by controlling $\varepsilon_1$) until failure occurred. As Figure 4 shows, $\sigma_1$ at failure generally increased at higher confining pressures ($\sigma_2 = \sigma_3$), yielding a strength criterion best fitted by a power function:

$$\sigma_1 = 164 + 18.7 \sigma_3^{0.79},$$

where the stress unit is MPa.

Specimens subjected to conventional triaxial loading failed along single or conjugate steep fault planes. We measured the average dip angle of the failure planes and found that they decreased gradually as the confining pressure was raised, from $\sim 70^\circ$ at $\sigma_3 = 0$ to $60^\circ$ at $\sigma_3 = 150$ MPa (Figure 5).
Figure 4. KTB amphibolite compressive strengths under different confining pressures based on conventional triaxial testing ($\sigma_2 = \sigma_3$) and the best fit curve representing the Mohr strength criterion in terms of $\sigma_1$ as a function of $\sigma_3$.

The bulk of our experiments was conducted under true triaxial compressive stress conditions ($\sigma_1 > \sigma_2 > \sigma_3 > 0$). For each level of $\sigma_3$ applied (0, 30, 60, 100 and 150 MPa), a series of tests were carried out in which different $\sigma_2$ magnitudes were maintained constant and $\sigma_1$ was increased to failure. All experimental results, including those from uniaxial and conventional triaxial tests are detailed in Table 2, and plotted in the $\sigma_1 - \sigma_2$ domain (Figure 6). The solid line in the plot represents the conventional triaxial strength criterion for $\sigma_2 = \sigma_3$ (shown also in Figure 4). Dashed lines are loosely drawn.

Figure 5. Fracture dip angle ($\theta$) as a function of confining pressure ($\sigma_2 - \sigma_3$) in conventional triaxial tests.

Figure 6. KTB amphibolite compressive strengths under true triaxial loading, plotted as the peak $\sigma_1$ versus $\sigma_2$ for different $\sigma_3$ magnitudes. The solid line represents the Mohr strength criterion (as in Figure 4). The dashed lines are loosely drawn curves indicating the trend of strength variation with $\sigma_2$ for given $\sigma_3$. 

$\sigma_1$ (MPa) $= 164 + 18.7 \sigma_3^{0.79}$
Table 2. True Triaxial Stress Conditions at Failure and
Average Fracture Angles $\theta^*$

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* Dashes indicate not measured.

curves showing the true triaxial strength trend for each family of tests for which $\sigma_1$ was held constant. They reveal a general increase in peak $\sigma_1$ with the magnitude of $\sigma_2$ until a plateau is reached as the intermediate principal stress attains magnitudes several times larger than $\sigma_3$. For the ranges of $\sigma_2$ used, there is no clear trend toward an eventual decrease in strength, as predicted by Wiebold and Cook [1968]; however, the trend is in accord with previous observations by Mogi [1971] and Haimson and Chang [2000].

Our results clearly demonstrate that true triaxial strength is typically higher than the corresponding conventional triaxial strength irrespective of the $\sigma_3$ level. The average percentage increase over the conventional triaxial equivalent (Table 2) reaches as high as ~50% at $\sigma_3$ > 30 MPa (for $\sigma_3$ levels of 200 to 300 MPa), 30% at $\sigma_3$ = 100 MPa (for $\sigma_3$ levels of 300 to 600 MPa), and 10% at $\sigma_3$ = 150 MPa (for $\sigma_3$ levels of 450 to 650 MPa). It is noteworthy that the effect of $\sigma_2$ on rock

strength appears to weaken at higher levels of $\sigma_3$ but remains significant. Thus conventional triaxial tests provide only a lower bound of strength for a given least principal stress in KTB amphibolite. Since in situ conditions of equal intermediate and least principal stresses are rare, criteria such as the Mohr-Coulomb (equation (3)) typically yield only conservative estimates of rock strength.

Our results support Vernik and Zoback's [1997] assertion that the Mohr-Coulomb criterion is not appropriate for computing in situ stresses from the stress-strength relationship at a borehole-borehole intersection, since the principal stresses there are highly differential. Our tests suggest that a correct evaluation of the crustal stress at the KTB site from the limit equilibrium between stress condition and rock resistance to failure at the borehole-borehole intersection requires the use of a true triaxial strength criterion [see also Brady et al., 1997].

Tested true triaxial specimens failed along one major steeply dipping fracture or fault (Figure 7) striking subparallel to $\sigma_3$ direction. Average fracture plane dip angles were measured and the results are listed in Table 2 and plotted in Figure 8. Under conditions of $\sigma_3$ larger than $\sigma_1$, the fracture plane was generally steeper than that under uniform confining pressure. Although angles plotted as a function of $\sigma_2$ show some scatter, the overall trend is that of dip steepening as the intermediate principal stress is raised [see also Mogi, 1971; Haimson and Chang, 2000]. This phenomenon provides additional strong evidence of the $\sigma_2$ effect on the mechanical behavior of the KTB amphibolite.

6. General Strength Criterion for the KTB Amphibolite

An important objective of this research was to find an all-inclusive strength criterion for the KTB amphibolite that will represent all the data shown in Figure 6 in the form of a unique relationship between the applied stresses at failure. The KTB amphibolite failed in brittle fashion along a fracture plane striking in the direction of $\sigma_3$, a similar behavior to that of the Dunham dolomite. Thus we considered first the criterion used by Mogi [1971] for that rock (equation (10)). We computed for each test the octahedral shear stress $\tau_{oct}$ and the mean normal stress acting on the failure plane $\sigma_{m,2}$, and plotted the experimental points in the new domain as shown in Figure 9. Test data fit strikingly well a monotonically increasing power function given by

$$\tau_{oct} = 1.77 \sigma_{m,2}^{0.86},$$

where the stress unit is MPa.

Equation (12) presents the relationship between all three principal compressive stresses at failure in KTB amphibolite and is clearly more general than Mohr-based criteria. It also demonstrates that Mogi's suggested strength criterion for rocks undergoing brittle failure can be extended to crystalline igneous and metamorphic rocks such as granite [Haimson and Chang, 2000] and amphibolite.

We also attempted to verify whether our experimental data could be fitted by the Nadai general criterion (equation (7)) or its linearized version (equation (8)). We plotted each of the stress conditions at failure in the $\tau_{oct}$ - $\sigma_{oct}$ domain as shown in Figure 10. Clearly, the scattered distribution of the experi-
mental points demonstrates that no unique function in this domain faithfully represents the true triaxial strength of the KTB amphibolite.

7. Stress-Strain Relationships Under True Triaxial Loading

As described above, several of the tested specimens were instrumented for continuous recording of all three principal strains. Stress-strain records of the major principal stress as a function of each of the three principal strains from the series of tests in which $\sigma_3$ magnitude was kept at 100 MPa, and $\sigma_2$ was varied from 100 MPa to 600 MPa, are shown in Figure 11. As a result of the loading path used in the tests, each stress-strain curve consists of three segments. In the first segment all three principal stresses were raised simultane-

ously (until $\sigma_3$ reached its predetermined level). In the second segment, $\sigma_1$, and $\sigma_2$ were increased simultaneously while $\sigma_3$ was kept constant until $\sigma_2$ had reached its preset magnitude. In the third and final segment, $\sigma_1$ alone was allowed to rise by controlling the principal strain in the $\sigma_1$ direction. To maintain consistency with the most common way of presenting stress-strain curves obtained from triaxial tests, we plotted the recorded data in the form of the maximum differential stress ($\sigma_1 - \sigma_3$) as a function of the three principal strains. Hence the first stage of the stress-strain curves is not shown in Figure 11. We note that during the simultaneous loading of both $\sigma_1$ and $\sigma_2$, all stress-strain curves were practically linear even when the preset value of $\sigma_2$ was as high as 600 MPa, and incremental strains $\Delta e_1$ and $\Delta e_2$ were both positive (contractions). However, in the final stage of loading (third segment of the stress-strain curve), only $\Delta e_1$ was positive, with the specimen extending in the other two principal directions.

Figure 11 shows that while the relationship $\sigma_1 - \sigma_3$ versus $\varepsilon_2$ is quasi-linear throughout the third segment of loading almost to the point of failure, the equivalent curve in the $\varepsilon_3$ direction is clearly nonlinear and indicates accelerated extension as $\sigma_1$ rises. This behavior suggests that the great majority of induced and reopened microcracks, leading to eventual brittle failure, is aligned with the $\sigma_1 - \sigma_3$ plane and opens up in the $\sigma_2$ direction.

Dilatancy, or the paradoxical volumetric expansion in rock subjected to rising compressive stress, is a measure of inelastic behavior prior to failure. Dilatancy has been correlated to internal microcracking responsible for expanding the volume and for leading eventually to the creation of the fracture plane [Brace et al., 1966]. We replotted the recorded stress-strain diagrams in the form of the maximum differential stress ($\sigma_1 - \sigma_3$) as a function of the volumetric strain and observed in all tests definite dilatancies. In an effort to gain some understanding of the effect of $\sigma_2$ on the extent of dilatancy, we selected two groups of tests in which $\sigma_1$ was kept constant (100 and 150 MPa, respectively) and plotted stress-volumetric strain for different $\sigma_2$ side by side. Figure 12 reveals that dilatancy is more pronounced for low $\sigma_2$ magnitudes and is considerably less evident at the higher levels of $\sigma_2$. Similar results were obtained by Takahashi and Koide [1989] in Shirahama sandstone.

The precise level of dilatancy onset, the inflection point where the stress-volumetric strain curve departs from the original elastic straight line and begins rotating backward, was somewhat ambiguous in the stress-strain curves shown in Figure 12 because of the gradual change in slope. To overcome this problem, we determined the dilatancy onset from a continuous plot of the derivative of ($\sigma_1 - \sigma_3$) with respect to volumetric strain as a function of ($\sigma_1 - \sigma_3$). Such a plot yields unambiguously the stress level at which the curve departs from a straight horizontal line, and that value was marked in Figure 12 as the point of dilatancy onset. We estimate the margin of error in the selection of the dilatancy onset by this method to be ±10 MPa.

Dilatancy onset in the KTB amphibolite generally increases with the magnitude of $\sigma_2$. Thus, for $\sigma_1 = 100$ MPa the onset of dilatancy rises steadily from as low as 38% of peak ($\sigma_1 - \sigma_3$) when $\sigma_2 = \sigma_3$ to 78% of the respective peak stress differential when $\sigma_3 = 600$ MPa (Figure 12a). A similar increase occurs in tests for which $\sigma_3$ is kept at 150 MPa (Figure 12b).

Analogous observations of dilatancy onset dependence on the magnitude of $\sigma_3$ were made by Mogi [1978] in Mizuho.
Figure 8. Fracture dip angle in true triaxial tests as a function of $\sigma_2$ for constant $\sigma_3$ magnitudes: (a) $\sigma_3 = 0$ MPa, (b) $\sigma_3 = 60$ MPa, (c) $\sigma_3 = 100$ MPa, and (d) $\sigma_3 = 150$ MPa.

trachyte and by Haimson and Chang [2000] in Westerly granite. Schock et al. [1973] reported diminishing dilatant behavior and an increase in the level of dilatancy onset at higher mean normal stresses in a granodiorite subjected to conventional triaxial loading. Our results confirm these findings and generalize them for true triaxial conditions.

Since the onset of dilatancy is considered an indicator of microcracking emergence, the significance of the trend shown in Figure 12 is that the intermediate principal stress extends the elastic range of the stress-strain behavior for a given $\sigma_3$ or, in other words, retards the beginning of the failure process.

8. Micromechanics Aspects of True Triaxial Compressive Failure

We studied aspects of the true triaxial fracture process by inspecting sections of failed specimens under a scanning electron microscope (SEM model JEOL JSM-6100). Sections

Figure 9. A general true triaxial strength criterion for the KTB amphibolite, based on all the experimental results shown in Figure 7 plotted as $\tau_{\text{int}}$ versus $\sigma_{m,2}$.

Figure 10. KTB amphibolite true triaxial test results plotted as $\tau_{\text{int}}$ versus $\sigma_{\text{int}}$, showing that the Nadai strength criterion does not apply in this rock.
Figure 11. Records of maximum differential stress ($\sigma_1 - \sigma_3$) as a function of strains in all three principal directions, obtained in four tests in which $\sigma_2$ was kept constant (100 MPa) but the intermediate principal stress varied as follows: (a) $\sigma_2 = 100$ MPa, (b) $\sigma_2 = 300$ MPa, (c) $\sigma_2 = 440$ MPa, and (d) $\sigma_2 = 600$ MPa.

Figure 12. Maximum differential stress ($\sigma_1 - \sigma_3$) variation with volumetric strain $\Delta V/V$, and the onset of dilation (see arrows), for different $\sigma_2$ and constant $\sigma_3$ magnitudes: (a) $\sigma_3 = 100$ MPa and (b) $\sigma_3 = 150$ MPa.
were cut along one of two planes, orthogonal to $\sigma_2$ direction (profiles), or orthogonal to $\sigma_1$ direction (cross sections). They were ground flat and polished down to 0.05 $\mu$m. The surface was then sputter coated with a 0.06-$\mu$m-thick carbon layer.

Figures 13-15 are SEM micrographs of sections of specimen profiles, with $\sigma_1$ acting vertically and $\sigma_2$ laterally. Under true triaxial stress conditions, microcracks develop along planes that are subparallel to $\sigma_1$, steeply dipping in the $\sigma_1$ direction, as suggested by Figure 13. However, the path along which cracks grow is affected by texture and structure. In one example (Figure 13a), stress-induced microcracks subparallel to $\sigma_1$ and extending through a zone of amphibole grains terminate at or are offset by a preexisting transverse fissure. Figure 13b depicts microcracks extending through cleavage planes in amphibole (light gray) and biotite (dark gray). Despite its high angle orientation with respect to the $\sigma_1$ direction, cleavage guides microcrack development, resulting in a step-like discontinuity.

The main fracture or fault upon compressive failure (Figure 7) appears to have formed from the coalescence of localized microcracks. A multitude of stress-induced microcracks subparallel to $\sigma_1$ direction on both sides of the fracture is exhibited in Figure 14. The steeply dipping fault also reflects some shear displacement inferred from the visible gouge within its opening and the offset noted in an amphibole grain. The grain appears to have been sheared by the fault (two arrows indicate the offset).

Close observation of microscale segments of the main fault reveals that it does not necessarily extend along a single plane as is often assumed. Two examples are shown in Figure 15. In one case (Figure 15a), the fault exhibits an en echelon array of three segments, each of which extends within the dark gray plagioclase and is terminated and offset by the light gray amphibole grains. In the other case (Figure 15b), two segments of the main fault that are parallel to each other but not aligned are linked along a biotite [001] cleavage. The fault aperture is not consistent and is widest in the upper right and the bottom left corner of the micrograph, where it passes through grain boundaries between amphibole and plagioclase.

Generally, micromechanical features described above are not significantly different from those observed in rocks subjected to conventional triaxial stresses [e.g., Tapponnier and

![Figure 13](image-url)

**Figure 13.** Two SEM micrographs showing the effect of structure on microcrack propagation: (a) a preexisting transverse fissure blocks or offsets stress-induced microcracks that are subparallel to $\sigma_1$ and (b) stress-induced microcrack extends in a step-like manner due to cleavages in amphibole (light gray) and biotite (dark gray) grains.
the fracture strike, although some are still randomly oriented. Thus a reduced blocky structure is created, rendering the main fracture considerably narrower. Microcracks become more and more aligned with the direction of $\sigma_2$ as the magnitude of the latter increases, and fragmentation all but disappears. As shown in Figures 16c and 16d, at $\sigma_2$ levels of 200 and 300 MPa the main fracture is clear of broken rock, resulting in a much narrower aperture than those in either Figure 16a or 16b. In addition, all four micrographs in Figure 16 provide further evidence that all visible microcracks are localized in the proximity of the main fault (within < 200 $\mu$m).

9. Summary and Conclusions

We conducted an extensive series of true triaxial compressive tests in KTB amphibolite using a newly designed polynaxial loading system that enables the application of three unequal principal stresses to a 19x19x38 mm prismatic specimen. The loading configuration is especially suited for testing brittle failure of hard crystalline rocks subjected to least principal stresses $\sigma_1$ of up to 400 MPa.

Our measurements resulted in individual strength criteria for each of five levels of $\sigma_2$ tested (0, 30, 60, 100, and 150 MPa) for which the intermediate principal stresses $\sigma_3$ varied between $\sigma_2 - \sigma_1$ and $\sigma_2 \geq 4.5 \sigma_1$. In all cases the trend in the peak principal stress $\sigma_1$ at failure was to increase with the magnitude of $\sigma_2$. At very high ($\sigma_1 - \sigma_2$) stress differential, $\sigma_1$ at failure appeared to reach a plateau, suggesting an upper limit for the magnitude of true triaxial strength. All true triaxial strength test results fit remarkably well a monotonically rising function in the domain of octahedral shear stress $\tau_{oct}$ versus mean normal stress $\sigma_{mean}$ on planes striking in the $\sigma_2$ direction. In the case of the KTB amphibolite the best fitting curve is a power function given by $\tau_{oct} = 1.77 \sigma_{mean}^{0.86}$.

Mohr-based failure criteria, which are all but universally used at present, ignore the effect of the intermediate principal stress and are typically obtained from conventional triaxial tests for which $\sigma_2 = \sigma_1$. Our true triaxial tests are clear affirmations that these criteria are only conservative estimates of rock strength, since they yield the lowest strength value for any given least principal stress.

In general, our results support Brady et al.'s [1997] hypothesis that a correct evaluation of the crustal stress in the KTB amphibolite from the limit equilibrium between stress condition and rock resistance to failure at borehole-breakout intersection requires the use of a true triaxial strength criterion. Upon failure, a steeply dipping major fault was created, which steepened further as the intermediate principal stress was raised (for the same least stress). This phenomenon provides additional evidence of the intermediate principal stress strengthening effect on the KTB amphibolite.

The intermediate principal stress has a strong effect on the deformability of the KTB amphibolite. This is best expressed in terms of the onset of dilatancy, which for the same $\sigma_2$, increases significantly with the magnitude of $\sigma_2$. Thus the intermediate principal stress appears to extend the elastic range of the stress-strain behavior for a given $\sigma_2$, which implies a retardation of the onset of the failure process.

We recorded the variation of the major principal stress with each of the principal strains and found that the only relation-

Figure 14. Section of the throughgoing main fracture or fault in a failed sample cuts across amphibole (light gray) and plagioclase (dark gray) grains and dips steeply in $\sigma_2$ direction. Shown also is a multitude of stress-induced microcracks localized on both sides of the fracture. White arrows within the fault indicate shear displacement manifested by a split amphibole grain.
Figure 15. Two examples of the tortuous path taken by the main fracture: (a) en echelon arrangement caused by amphibole grains (light gray) offsetting fracture tip and (b) a biotite grain (center of micrograph) links along its cleavage two parallel but not aligned segments of the main fracture.
ship that is clearly nonlinear is that between σ₁ and ε₃, inferring accelerated extension in the σ₃ direction as σ₁ rises. This behavior suggests that the great majority of induced and re-opened microcracks, leading to eventual brittle failure, is aligned with the σ₁ - σ₃ plane.

Micromechanical behavior upon brittle failure under true triaxial compression was explored using the SEM. The main fault, although planar when viewed from afar, develops microscopically along a rather tortuous path, mainly as a result of encountering minerals that force offsets and en echelon arrangements. Micrographs of cross sections in the σ₂ - σ₃ plane demonstrate that in true triaxial tests, microcracks localize in the very close proximity to the major fault. These micrographs also reveal the variation in microcrack alignment from largely random when σ₂ = σ₃ to one parallel to σ₂ direction as the differential stress (σ₂ - σ₁) increases.

Acknowledgments. This research was sponsored by the National Science Foundation grant EAR-9418738. We are indebted to Ulrich Harms, GFZ Potsdam, for his assistance with procuring samples from the KTB core. Comments made and improvements suggested by Teng-fong Wong and an anonymous reviewer greatly enhanced the quality of the paper.

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(Received November 16, 1999; revised May 15, 2000; accepted May 19, 2000.)